

Chapter 10

Analysis of Variance

The Completely Randomized Design: Single Factor

Definition 10.7 The *completely randomized design* is a design in which k treatments are randomly assigned to the experimental units or in which independent random samples of experimental units are selected for each treatment.

The objective of a completely randomized design is usually to compare the treatment means. If we denote the true, or population, means of the k treatments as $\mu_1, \mu_2, \dots, \mu_k$ then we will test that the population means are all equal against the alternative that at least two of the treatment means differ.

For all ANOVA problems the hypotheses will be

$$H_0 : \mu_1 = \mu_2 = \dots = \mu_k \text{ (All the } k \text{ treatment means are equal)}$$

$$H_a : \text{At least 2 of the } k \text{ treatment means differ}$$

Example 10.4 Conducting an ANOVA F -Test – Comparing Golf Ball Brands

Problem: Suppose the USGA wants to compare the mean distances reached of four different brands of golf balls struck with a driver. A completely randomized design is employed, with Iron Byron, the USGA's robotic golfer, using a driver to hit a random sample of 10 balls of each brand in a random sequence. The distance is recorded for each hit, and the results are shown in Table 10 – 1. Perform a hypothesis test to compare the mean distances of the four brands of golf balls. Use $\alpha = 0.10$. Include the test statistic, p -value and state the appropriate conclusion.

Table 10 – 1

Distance for
Each Hit

Brand A	Brand B	Brand C	Brand D
251.2	263.2	269.7	251.6
245.1	262.9	263.2	248.6
248.0	265.0	277.5	249.4
251.1	254.5	267.4	242.0
260.5	264.3	270.5	246.5
250.0	257.0	265.5	251.3
253.9	262.8	270.7	261.8
244.6	264.4	272.9	249.0
254.6	260.6	275.6	247.1
248.8	255.9	266.5	245.9

Solution:

1. We begin by entering all data into the calculator. Enter each brand into its own list. In this example we will use L1, L2, L3 and L4. See Figure 10 – 1.

L2	L3	L4	4
264.3	270.5	246.5	
257	265.5	251.3	
262.8	270.7	261.8	
264.4	272.9	249	
260.6	275.6	247.1	
255.9	266.5	245.9	
-----	-----	-----	
L4(11) =			

Figure 10 – 1

2. After you have entered all the data, press **[STAT]** and arrow over to the TESTS menu. Arrow up to **H:ANOVA**(and press **[ENTER]**. See Figure 10 – 2.

```

EDIT CALC TESTS
B:1-PropZInt...
C:x²-Test...
D:x²GOF-Test...
E:2-SampFTest...
F:LinRegTTest...
G:LinRegTInt...
H:ANOVA(
    
```

Figure 10 – 2

3. Press **[ENTER]**. The screen will appear as in Figure 10 – 3. For ANOVA you simply enter the lists where you placed the data. For this problem enter L1, L2, L3, L4. See Figure 10 – 4.



Figure 10 – 3

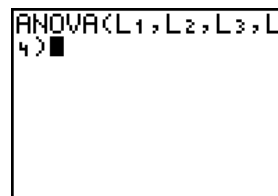


Figure 10 – 4

4. Press **[ENTER]**. To see all the results you must arrow down. See figures 10– 5 and 10 – 6.

```

One-way ANOVA
F=43.98874592
P=3.973108E-12
Factor
df=3
SS=2794.38875
MS=931.462917
    
```

Figure 10 – 5

```

One-way ANOVA
MS=931.462917
Error
df=36
SS=762.301
MS=21.1750278
SxP=4.60163316
    
```

Figure 10 – 6

All values requested can be read directly from the screens in Figure 10 – 5 and 10 – 6.

The test statistic is $F = 43.98874592$.

The p-value is $3.97108E-12 = 0.000000000003973108$.

As with any hypothesis test, we will reject H_0 in favor of H_a when the p-value $< \alpha$. Since the p-value is extremely small, we will reject H_0 . We have sufficient evidence that at least two of the brands of golf balls have different mean distances traveled when struck with a driver.

Exercise 10.26 Completing an ANOVA Table and Conducting an ANOVA F-test

Problem: The data in the Table 10 – 2 resulted from an experiment that utilized a completely randomized design.

- a) Complete an ANOVA table.
- b) Test the null hypothesis that $\mu_1 = \mu_2 = \mu_3$ against the alternative that at least two of the means differ. Use $\alpha = 0.01$.

Table 10 – 2

Treatment 1	Treatment 2	Treatment 3
3.9	5.4	1.3
1.4	2.0	0.7
4.1	4.8	2.2
5.5	3.8	
2.3	3.5	

Solution:

- a)
 1. We begin by entering all data into the calculator. Enter each treatment into its own list. In this example we will use L1, L2, and L3. See Figure 10 – 7. Note: In ANOVA it is not necessary that the lists be of equal length.

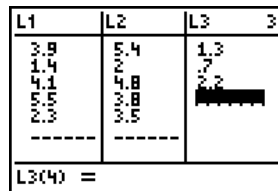


Figure 10 – 7

- 2. After you have entered all the data, press **[STAT]** and arrow over to the TESTS menu. Arrow up to **H:ANOVA(** and press **[ENTER]**. See Figure 10 – 8.

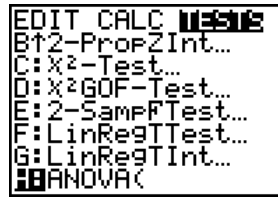


Figure 10 – 8

3. Press **ENTER**. The screen will appear as in Figure 10 – 9. For ANOVA you simply enter the lists where you placed the data. For this problem enter L1, L2, L3. See Figure 10 – 10.



Figure 10 – 9

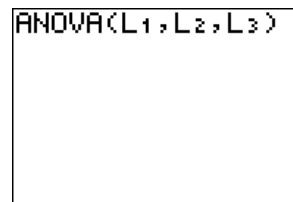


Figure 10 – 10

4. Press **ENTER**. To see all the results you must arrow down. See figures 10–11 and 10 – 12.

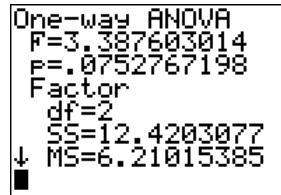


Figure 10 – 11

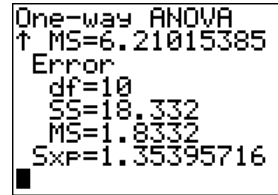


Figure 10 – 12

All values requested for the ANOVA table can be read directly from the screens as the calculator uses standard notation. In Figure 10 – 11 are the values for the treatment which is also called the factor. These values go in the row entitled “treatments” in the ANOVA table. In Figure 10 – 12 are the values for the error which appear in the row entitled “error” in the ANOVA table. These values are used to complete the ANOVA table in Figure 10 – 13.

Source	df	SS	MS	F
Treatments	2	12.420	6.2102	3.3876
Error	10	18.332	1.8332	
Total	12	30.752		

Figure 10 – 13

b) To test the null hypothesis that $H_0 : \mu_1 = \mu_2 = \mu_3$ against the alternative that at least two of the means differ, we see from the Figure 10 – 11 that the p-value is 0.075 which is not less than $\alpha = 0.01$ therefore we do not reject H_0 . We do not have sufficient evidence that at least two of the population means differ.