Chapter 9
Inferences Based on Two Samples

Comparing Two Population Means: Independent Samples

Example 9.1 A Large-Sample Confidence Interval for ($\mu_1 - \mu_2$):
Comparing Mean Weight Loss for Two Diets

Problem: A dietician has developed a diet that is low in fats, carbohydrates, and cholesterol. Although the diet was initially intended to be used by people with heart disease, the dietician wishes to examine the effect this diet has on the weights of obese people. Two random samples of 100 obese people each are selected and one group of 100 placed on the low-fat diet. The other 100 people are places on a diet that contains approximately the same quantity of food, but is not low in fats, carbohydrates or cholesterol. For each person the amount of weight lost (or gained) in a three-week period is recorded. Form a 95% confidence interval for the difference between the population mean weight loss for the two diets.

Solution: On the calculator we will go to the **STAT TESTS** menu once again.

There are 3 different 2-Sample Intervals and 3 different 2-Sample Tests under the **TESTS** Menu. Because of the large sample sizes found in this problem we will use the 2-Sample Z-Interval.

Let μ_1 denote the mean weight loss for the low-fat diet and μ_2 denote the mean weight loss for the regular diet.

1. Begin by entering all data for the low-fat diet into List1 & all the data for the regular diet into list2. Then press STAT and arrow over to the TEST menu. See Figure 9 – 1.


```
EDIT CALC TESTS
4↑2-SampTTest…
5:1-PropZTest…
6:2-PropZTest…
7:ZInterval…
8:TInterval…
9⌷2-SampZInt…
0↓2-SampTInt…
```
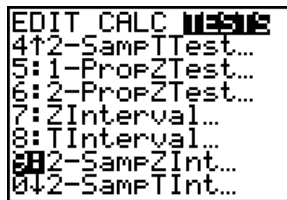
Figure 9 – 1

2. Number 9 is the two-sample Z-interval so press 9 or arrow down to 9 and press ENTER. See Figure 9 – 1.

3. Highlight Data and press ENTER.

4. Notice that we choose Data because we have entered the data into List 1 and List 2. However, we still must enter the standard deviation into the calculator. If the standard deviation is not given in the problem, as is our case, you must first run "1-var stats" on each list to obtain sample the standard deviations. The sample standard deviations will be used to estimate the population variances. For this problem $s_1 = 4.67$ and $s_2 = 4.04$ so enter 4.67 as the estimate for σ_1 and then enter 4.04 as the estimate for σ_2.
See Figure 9 – 2.

5. Enter L_1 as List 1 by pressing [2nd] L1 [ENTER]. See Figure 9 – 2.

6. Enter L_2 as List 2 by pressing [2nd] L2 [ENTER]. See Figure 9 – 2.

7. Enter Freq1 and Freq2 as 1. Recall the TI-83/84 Plus goes into Alpha mode for Frequencies so press [ALPHA] [1]. See Figure 9 – 2.

8. Enter the C-level as 0.95 and press [ENTER]. See Figure 9 – 3.

Figure 9 – 2   Figure 9 – 3

9. Highlight Calculate and press [ENTER]. Your screen should appear as in Figure 9 – 4.

Figure 9 – 4

As can be seen from the screen in Figure 9 – 4, the 95% CI of the difference between the population means is 0.69072 to 3.1293. Thus, we are 95% confident the mean weight loss for the low-fat diet is between 0.69072 and 3.1293 pounds more than the mean weight loss for the other diet.

Examples 9.2 & 9.3 A Large-Sample Test for ($\mu_1 - \mu_2$):
Comparing Mean Weight Loss for Two Diets

Problem: Refer to the study of obese people on a low-fat diet and a regular diet presented in example 9.1. Conduct a hypothesis test to compare the mean weight loss for the two different diets. Use $\alpha = 0.05$.

Solution: Once again we go to the **STAT TESTS** Menu, and for this problem we will choose a 2-Sample Z-Test as requested by the problem. We will test the alternative hypothesis that the two means are not equal, and use alpha = .05 as given in the problem.

Again, let μ_1 denote the mean weight loss for the low-fat diet and μ_2 denote the mean weight loss for the regular diet. Then the null and alternative hypotheses are:

H_0: $\mu_1 = \mu_2$ (mean weight loss is the same)

H_a: $\mu_1 \neq \mu_2$ (mean weight loss is different)

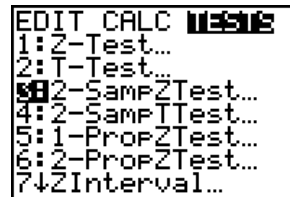1. Press $\boxed{\text{STAT}}$ and arrow over to the TEST menu. See Figure 9 – 5.


```
EDIT CALC TESTS
1:Z-Test…
2:T-Test…
3:2-SampZTest…
4:2-SampTTest…
5:1-PropZTest…
6:2-PropZTest…
7↓ZInterval…
```

Figure 9 – 5

2. Number 3 is the two-sample Z-test so press $\boxed{3}$ or arrow down to 3 and press $\boxed{\text{ENTER}}$. See Figure 9 – 5.

3. Once again you must choose Data or Stats . For our problem we choose Stats because we will enter the summary statistics ourselves. Therefore, highlight Stats and press $\boxed{\text{ENTER}}$.

4. The sample standard deviations will be used to estimate the population variances. For this problem $s_1 = 4.67$ and $s_2 = 4.04$ so enter 4.67 as the estimate for σ_1 and press $\boxed{\text{ENTER}}$. Then enter 4.04 as the estimate for σ_2 and press $\boxed{\text{ENTER}}$. See Figure 9 – 6.

5. Enter $\bar{x}_1$ as 9.31 and press $\boxed{\text{ENTER}}$. See Figure 9 – 6.

6. Enter n_1 as 100 and press $\boxed{\text{ENTER}}$. See Figure 9 – 6.

7. Enter $\bar{x}_2$ as 7.40 and press $\boxed{\text{ENTER}}$. See Figure 9 – 6.

8. Enter n_2 as 100 and press ENTER. See Figure 9 – 6.

9. Highlight the appropriate hypothesis and press ENTER. We will highlight $\neq \mu_2$ for our example.   See Figure 9 – 7.


```
2-SampZTest
 Inpt:Data Stats
 σ1:4.67
 σ2:4.04
 x̄1:9.31
 n1:100
 x̄2:7.4
↓n2:100
```
Figure 9 – 6

```
2-SampZTest
↑σ2:4.04
 x̄1:9.31
 n1:100
 x̄2:7.4
 n2:100
 μ1:≠μ2 <μ2 >μ2
 Calculate Draw
```
Figure 9 – 7

10. Highlight Calculate and press ENTER. Your screen will appear as in Figure 9 – 8.

```
2-SampZTest
 μ1≠μ2
 z=3.093122479
 p=.0019807601
 x̄1=9.31
 x̄2=7.4
↓n1=100
■
```
Figure 9 – 8

As can be seen from the screen in Figure 9 – 8, the test statistic is z = 3.09 and the p-value is 0.002 which is less than the significance level of 0.05 therefore we reject Ho. There is sufficient evidence the two mean weight of the low-fat diet and the regular diet are not equal at α = .05.

Example 9.4 A Small-Sample Confidence Interval for $(\mu_1 - \mu_2)$:
Comparing Two Methods of Teaching

Problem: Suppose you wish to compare a new method of teaching reading to "slow learners" with the current standard method. You decide to base your comparison on the results of reading test given at the end of learning period of six months. Of a random sample of 22 "slow learners", 10 are taught by the new method and 12 are taught by the standard method. All 22 children are taught by qualified instructors under similar conditions for the designated six-month period. The results of the reading test at the end of the period are given in Table 9 – 1. Use a 95% confidence interval to estimate the true mean difference between the test scores for the new method and standard method

Table 9 – 1 Reading Test Scores	New Method				Standard Method			
	80	80	79	81	79	62	70	68
	76	66	71	76	73	76	86	73
	70	85			72	68	75	66

Solution: For this problem we will go back to the **STAT TESTS** Menu, but this time we will choose a **2-Sample T-Interval** as the sample size dictates.

Let μ_1 denote the mean test score for the new method and let μ_2 denote the mean test score for the standard method.

1. Begin by entering all data for the new method into List1 & all the data for the standard method into list2. See Figure 9 – 9.
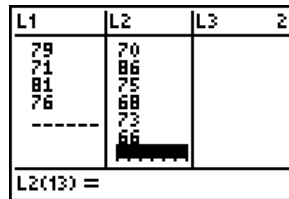

Figure 9 – 9

2. Then press STAT and arrow over to the TEST menu. Number 0 is the two-sample t-interval so press 0 or arrow down to 0 and press ENTER. See Figure 9 – 10.
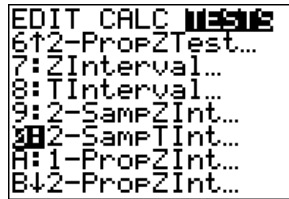

Figure 9 – 10

3. Since we have entered the data into lists, we will highlight Data and press ENTER.

4. Enter L_1 as List 1 by pressing 2nd L1 ENTER. See Figure 9 – 11.

5. Enter L_2 as List 2 by pressing 2nd L2 ENTER. See Figure 9 – 11.

6. Enter Freq1 and Freq2 as 1. Recall the TI-83/84 Plus goes into Alpha mode for Frequencies so press ALPHA 1. See Figure 9 – 11.

7. Enter the C-level as 0.95 and press ENTER.

8. When asked whether to pool the sample variances together or not, highlight Yes and press ENTER to select a pooled interval. Your screen should appear as in Figure 9 – 11. We choose **Yes,** since based on the F-test we cannot reject the claim the two sample variances are equal.

```
2-SampTInt
 Inpt:Data Stats
 List1:L₁
 List2:L₂
 Freq1:1
 Freq2:1
 C-Level:.95
↓Pooled:No Yes
```
Figure 9 – 11

9. Highlight Calculate and press ENTER. You will need to scroll down to see both screens. The first screen should appear as in Figure 9 – 12 and when you scroll down the screen will appear as in Figure 9 – 13.

```
2-SampTInt
 (-1.399,9.5327)
 df=20
 x̄₁=76.4
 x̄₂=72.33333333
 Sx₁=5.83476173
↓Sx₂=6.34369169
■
```
Figure 9 – 12

```
2-SampTInt
 (-1.399,9.5327)
↑Sx₁=5.83476173
 Sx₂=6.34369169
 SxP=6.11991285
 n₁=10
 n₂=12
■
```
Figure 9 – 13

As seen from the screens in Figure 9 – 12 and 9 – 13, the 95% CI for the true mean difference is **-**1.399 to 9.5327. Thus, we are 95% confident the mean test score for the new method could be as much as 1.399 points less than to as much as 9.5327 greater than the mean test score of the standard method.

Comparing Two Population Means: Paired Difference Experiment

Example 9.5 Confidence Interval for μ_d :
Comparing Mean Salaries of Males and Females
Problem: An experiment is conducted to compare the starting salaries of male and female college graduates who find jobs. Pairs are formed by choosing a male and a female with the same major and similar grade point averages (GPA). Suppose a random sample of 10 pairs is formed in this manner and the starting salary of each person is recorded. The results are shown in Table 9 – 2. Compare the mean starting salary for males with the mean starting salary for females using a 95% confidence interval.

	Pair	Male	Female	Difference = Male – Female
	1	$29,300	$28,800	$500
Table 9 – 2	2	41,500	41,600	-100
Annual Salaries	3	40,400	39,800	600
for Matched Pairs	4	38,500	38,500	0
of College	5	43,500	42,600	900
Graduates	6	37,800	38,000	-200
	7	69,500	69,200	300
	8	41,200	40,100	1100
	9	38,400	38,200	200
	10	59,200	58,500	700

Solution:

1. To perform a Paired Difference Confidence Interval on the calculator enter the data into List 1 and List 2. Enter all the data for the males into List 1 and enter all the data for female into List 2. The next step is to create List 3 which will be the difference between the two lists, namely (List 1 - List 2).   For matched pair calculations you MUST enter the data in the EXACT order it is given. See Figure 9 – 14.

2. After you have entered all the data Arrow up and highlight the **L3** itself (not the top line of the data). See Figure 9 – 14.

Figure 9 – 14

Figure 9 – 15

3. Pressing [2nd] L1 then the subtraction sign [-] then press [2nd] L2. You will see the expression scroll across the bottom of your screen as in Figure 9 – 15.

4. Press [ENTER]. List 3 will be created as you press [ENTER]. List 3 is the difference between List 1 and List 2. See Figure 9 – 16.

Figure 9 – 16

5. Press STAT and arrow over to the TESTS menu. Number 8 is the TInterval so either press 8 or arrow down to 8 and press ENTER. See Figure 9 – 17.

Figure 9 – 17

6. Highlight Data and press ENTER. See Figure 9 – 18.

7. Press 2nd L3 to enter List 3. Notice we choose data and we are using List 3 because we are working with the set of paired differences which are in L3. See Figure 9 – 18.

8. Set the Freq: as 1. See Figure 9 – 18.

9.   Set the C-Level as 0.95. Your screen should appear as in Figure 9 – 18.

Figure 9 – 18

10. Highlight Calculate and press ENTER. Your answer will be displayed as in Figure 9 – 19.

Figure 9 – 19

As can be seen from the screen in Figure 9 – 19, the 95% CI is 89.096 to 710.9. Thus, we are 95% confident the true mean difference between the starting salaries of males and females fall between $89 and $710 when rounded to the nearest dollar.

Comparing Two Population Proportions: Independent Samples

Example 9.6 A Large-Sample Test about ($p_1 – p_2$):
Comparing Fractions of Smokers for Two Years
Problem: In the past decade, intensive antismoking campaigns have been sponsored by both federal and private agencies. Suppose the American Cancer Society randomly sampled 1500 adults in 2000 and then sampled 1750 adults in 2010 to determine whether there was evidence that the percentage of smokers had decreased. The results of the two sample surveys are shown in Table 9 – 3. Do these data indicate that the fraction of smokers decreased over this 10-year period? Use $\alpha = 0.05$.

		Year 2000	Year 2010
Table 9 – 3 Results of Smoking Survey	Number of Smokers	$x_1 = 555$	$x_2 = 578$
	Sample Size	$n_1 = 1500$	$n_2 = 1750$

Solution: For the 2-Sample Proportion test we will again use the **STAT TESTS** menu. The given information is: $n_1 = 1500$, $x_1 = 555$, $n_2 = 1750$, $x_2 = 578$.

Let p_1 be the percentage of all adults who smoked in 2000 and let p_2 be the percentage of all adults who smoked in 2010. Then the null and alternative hypotheses are:

H_0: ($p_1 – p_2$) = 0 (the proportion of smokers has not decreased)
H_a: ($p_1 – p_2$) > 0 (the proportion decreased during the 10-year period)

1. Press $\boxed{\text{STAT}}$ and arrow over to the TESTS menu. See Figure 9 – 20.

2. Press $\boxed{6}$ or arrow down to **6:2-PropZTest** and press $\boxed{\text{ENTER}}$. See Figure 9 – 20.
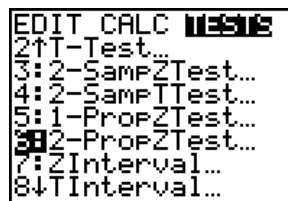
Figure 9 – 20

3. Enter your x_1, here 555, and press $\boxed{\text{ENTER}}$. See Figure 9 – 21.

4. Enter your n_1, here 1500, and press $\boxed{\text{ENTER}}$. See Figure 9 – 21.

5. Enter your x_2, here 578, and press $\boxed{\text{ENTER}}$. See Figure 9 – 21.

6. Enter your n_2, here 1750, and press $\boxed{\text{ENTER}}$. See Figure 9 – 21.

7. Highlight your alternative hypothesis and press $\boxed{\text{ENTER}}$. The inequality must match your H_a so we will use >p2 for our example. Your screen should appear as in Figure 9 – 21.

```
2-PropZTest
 x1:555
 n1:1500
 x2:578
 n2:1750■
 p1:≠p2 <p2 >p2
 Calculate Draw
```
Figure 9 – 21

8. Highlight Calculate and press $\boxed{\text{ENTER}}$. Your screen should appear as in Figure 9 – 22.

```
2-PropZTest
 p1>p2
 z=2.368523545
 p=.0089296078
 p̂1=.37
 p̂2=.3302857143
↓p̂=.3486153846
```
Figure 9 – 22

As can be seen from the screen in Figure 9 – 22, the test statistic is z = 2.369 and the p-value is 0.0089 which is less than the significance level of 0.05 therefore we reject Ho. There is sufficient evidence at to conclude that the proportion of adult smokers has decreased.

Comparing Two Population Variances: Independent Sampling (Optional)

Example 9.10 An F-Test Application: Comparing Weight Variations in Mice
Problem: An experimenter wants to compare the metabolic rates of white mice subjected to different drugs. The weights of the mice may affect their metabolic rates; thus, the experimenter wishes to obtain mice that are relatively homogeneous with respect to weight. Five hundred mice will be needed to complete the study. Currently 13 mice from supplier 1 and another 18 mice from supplier 2 are available for comparison. The experimenter weighs these mice and obtains the data obtained in Table 9 – 4. Do these data provide sufficient evidence to indicate difference in the variability of weights of mice obtained from the two suppliers? Use $\alpha = 0.10$.

	Supplier 1		Supplier 2	
	4.23	3.75	4.14	4.11
	4.01	4.19	4.17	4.21
	4.29	4.41	4.15	4.25
	4.35	4.52	4.26	4.31
	4.06	4.37	4.35	4.05
	4.05	4.21	4.20	4.02
	4.15		4.05	4.12
			4.25	4.28
			4.32	4.14

Table 9 – 4
Weights
(in ounces) of
Experimental Mice

Solution: We will once again use the **STAT TESTS** Menu. To perform an F-Test on the calculator put the data from Supplier 1 into list 1 and supplier 2 into list 2. Let σ_1^2 be the variance of weights for all experimental mice from supplier 1 and let σ_2^2 be the variance of the weights for all experimental mice from supplier 2. Then the null and alternative hypotheses are:

$$H_0 : \sigma_1^2 = \sigma_2^2 \text{ (Variability of weights is the same for both suppliers)}$$
$$H_a : \sigma_1^2 \neq \sigma_2^2 \text{ (Variability of weights differs for the two suppliers)}$$

1. Press $\boxed{\text{STAT}}$ and arrow over to the TESTS menu. See Figure 9 – 23.

2. Procedure D is the two sample F-test so press $\boxed{\text{ALPHA}}$ D or arrow down to **D:2-SampFTest** and press $\boxed{\text{ENTER}}$. See Figure 9 – 23.
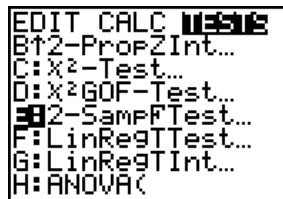
Figure 9 – 23

3. Highlight Data and press $\boxed{\text{ENTER}}$. See Figure 9 – 24.

4. Enter L_1 for List 1 and L_2 for List 2 by pressing $\boxed{\text{2nd}}$ L1 $\boxed{\text{ENTER}}$ $\boxed{\text{2nd}}$ L2.
See Figure 9 – 24.

5. Enter 1 for Freq1 and Freq2. (Recall the TI-83/84 Plus goes into Alpha mode for Frequencies so press $\boxed{\text{ALPHA}}$ $\boxed{1}$.) See Figure 9 – 24.

6. Highlight the appropriate alternative hypothesis and press ENTER. For this example we will highlight $\neq \sigma_2$. Your screen should appear as in Figure 9 – 24.

Figure 9 – 24

7. Highlight Calculate and press ENTER. Your screen will appear as in Figure 9 – 25.

Figure 9 – 25

As can be seen from the screen in Figure 9 – 25, the test statistic is F= 4.237 and the p-value is 0.007. Since the p-value is less than the significance level of 0.10, we reject H_0. At the $\alpha = 0.10$ level there is sufficient evidence that the two population variances are not equal.

Example 9.12 Checking the Assumption of Equal Variances

Problem: In Example 9.4, we used the two-sample t-statistic to compare the mean reading scores of two groups of "slow learners" who had been taught to read by two different methods. The use of the t-statistic was based on the assumption that the population variances of the test scores were equal for the two methods. The data is presented again in Table 9 – 5. Conduct a test of hypotheses to check this assumption at $\alpha = 0.10$.

	New Method				Standard Method			
Table 9 – 5 Reading Test Scores	80	80	79	81	79	62	70	68
	76	66	71	76	73	76	86	73
	70	85			72	68	75	66

Solution: Let σ_1^2 and σ_2^2 denote, respectively, the population standard deviations of reading scores for the new method and the standard method. The null and alternative hypotheses are:

H_0: $\sigma_1^2 = \sigma_2^2$ (standard deviations of reading scores are the same.)

H_a: $\sigma_1^2 \neq \sigma_2^2$ (standard deviations of reading scores are different.)

1. First enter the data for each method into its own List. See Figure 9 – 26.

Figure 9 – 26

2. Press $\boxed{\text{STAT}}$ and arrow over to the TESTS menu. Procedure E is the two sample F-test so press $\boxed{\text{ALPHA}}$ E or arrow down to E:2-SampFTest and press $\boxed{\text{ENTER}}$. See Figure 9 – 27.

Figure 9 – 27

3. Highlight Data and press $\boxed{\text{ENTER}}$. See Figure 9 – 28.

4. Enter L_1 for List 1 and L_2 for List 2 by pressing $\boxed{\text{2nd}}$ L1 $\boxed{\text{ENTER}}$ $\boxed{\text{2nd}}$ L2.
See Figure 9 – 28.

5. Enter 1 for Freq1 and Freq2. (Recall the TI-83/84 Plus goes into Alpha mode for Frequencies so press $\boxed{\text{ALPHA}}$ $\boxed{1}$.) See Figure 9 – 28.

6. Highlight the appropriate alternative hypothesis and press $\boxed{\text{ENTER}}$. For this example we will highlight $\neq \sigma_2$. Your screen should appear as in Figure 9 – 28.

Figure 9 – 28

7. Highlight Calculate and press ENTER. Your screen will appear as in Figure 9 – 29.


```
2-SampFTest
 σ1≠σ2
 F=.8459839357
 p=.8147505689
 Sx1=5.83476173
 Sx2=6.34369169
↓x̄1=76.4
■
```

Figure 9 – 29

As can be seen from the screen in Figure 9 – 29, the test statistic is F=0.8460 and the p-value = 0.8148 which is not less than .10, therefore we do not reject Ho. At the $\alpha = 0.10$ level, we cannot conclude there is sufficient evidence that there is a difference in the population variances of the two reading methods.