Chapter 8
Inferences Based on a Single Sample

Test of Hypothesis about a Population Mean: Normal (Z) Statistic

Examples 8.3 & 8.4 Setting up and Conducting a Hypothesis Test for μ – Mean Drug Response Time

Problem: The effect of drugs and alcohol on the nervous system has been the subject of considerable research. Suppose a research neurologist is testing the effect of a drug on the response time by injecting 100 rats with a unit does of the drug, subjecting each rat to a neurological stimulus, and recording its response time. The neurologist knows that the mean response times for rats not injected with the drug (the "control" mean) is 1.2 seconds. She wishes to test whether the mean response time for the drug-injected rats differs from 1.2 seconds. After performing the experiment, she determines that $\bar{x} = 1.05$ seconds and s = 0.5 seconds. Set up and conduct the hypothesis test using $\alpha = 0.01$.

Solution: Working a Test of Hypothesis on the calculator is very different and much less complicated than doing the computations by hand. While the methods are different the end results are ALWAYS the same. For **all** Tests of Hypothesis you will go to the **STAT TESTS** Menu.

For this problem we set up our Hypothesis as:

$$H_0 : \mu = 1.2$$
$$H_a : \mu \neq 1.2$$

Remember: The equal sign <u>always</u> goes with H_0 and you <u>always</u> test H_a.

To Compute the value of the test statistic and the p-value

There are two ways to perform a one-sample z-test using the TI-83. One method is useful if you have the data values but have not computed the sample mean. The other method is useful if you have the summary statistics. In this example we were given $\bar{x} = 1.05$ seconds, s = 0.5 seconds and n=100 so we will use the summary statistics.

1. Press [STAT] and arrow over to the TEST menu. See Figure 8 – 1.

2. Number 1 is the Z-Test so press [1] or press [ENTER]. See Figure 8 – 1.

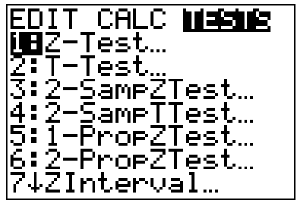

Figure 8 – 1

3. Highlight Stats and press ENTER. Your screen will appear as in Figure 8 – 2.

4. Enter μ_0 as 1.2 and press ENTER. See Figure 8 – 2.

5. Since we do not have σ we will use s to estimate it so enter 0.5 and press ENTER. See Figure 8 – 2.

6. Enter the sample mean, 1.05 and press ENTER. See Figure 8 – 2.

7. Enter the sample size, 100, and press ENTER. See Figure 8 – 2.

8. Highlight the appropriate alternative hypothesis and press ENTER. We will highlight $\neq \mu_0$ for our example. Your screen should appear as in Figure 8 – 2.

9. Highlight Calculate and press ENTER. Your screen will appear as in Figure 8 – 3.

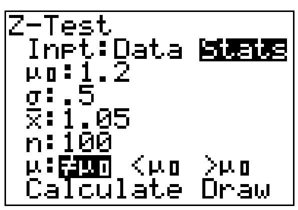

Figure 8 – 2

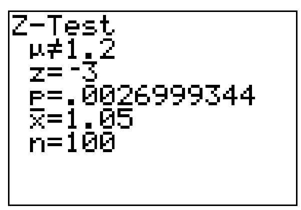

Figure 8 – 3

Note that Figure 8 – 3 displays the test statistic and the p-value. Here the test statistic is given as z = -3 and the *p*-value is 0.0027 when rounded to four decimal places.

Note: Your answers for the test statistic and p-value may vary slightly from the answers in the textbook because the TI-83/84 Plus does not round the mean when computing the test statistic and it does not round the test statistic to compute the p-value. However, your final conclusion will always be the same.

Remember if the p-value is less than the level of significance then we reject H_0**.** Otherwise do not reject H_0. There is no other Rule. It will always work.

In the example, $0.0027 < 0.01$ we will reject H_0. In conclusion we have sufficient evidence the mean response time of the drug-injected rats differs from the control mean.

In Figure 8 – 3 we see that the screen displays additional information. In particular,

The top line reminds you what test you are performing
The second line states what you are testing(the H_a)
The third line is the test statistic
The fourth line is the p-value
The fifth line is the sample mean
The bottom line is the sample size

Test of Hypothesis about a Population Mean: Student's t-Statistic

Examples 8.7 & 8.8 A Small-Sample Test for μ – Does a New Engine Meet Air-Pollution Standards?

Problem: A major car manufacturer wants to test a new engine to determine whether it meets new air-pollution standards. The mean emission of all engines of this type must be less than 20 parts per million of carbon. Ten engines are manufactured for testing purposes, and the emission level of each is determined. The data (in parts per million) are listed in Table 8 – 1. Do the data supply sufficient evidence to allow the manufacturer to conclude that this type of engine meets the pollution standard? Use $\alpha = 0.01$.

Table 8 – 1 Emission Levels

15.6	22.5	16.4	19.6	12.7
16.2	20.5	19.4	17.9	14.9

Solution:

Let μ denote the mean emission level of all engines of this type. Then the null and alternative hypotheses are

H_0: μ = 20 (mean emission level is 20 ppm).

H_a: μ < 20 (mean emission level is less than 20 ppm).

To compute the value of the test statistic and the p-value:

There are two ways to perform a one-sample t-test using the TI-83. One method is useful if you have the data values but have not computed the sample mean. We will demonstrate this method since we have the data.

1. Enter the data into List 1 (L1). See Figure 8 – 4.

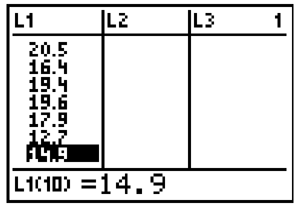

Figure 8 – 4

2. Press [STAT] and arrow over to the TEST menu. See Figure 8 – 5.

3. Number 2 is the T-test so press [2] or arrow down to 2 and press [ENTER]. See Figure 8 – 5.

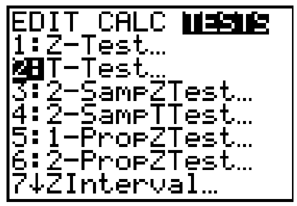

Figure 8 – 5

4. Highlight Data and press [ENTER]. See Figure 8 – 6.

5. Enter μ_0 as 20 and press [ENTER]. See Figure 8 – 6.

6. Enter List as List 1 by pressing [2nd] L1. See Figure 8 – 6.

7. Enter Freq: as 1. Recall the TI-83/84 Plus goes into Alpha mode for Frequencies so press [ALPHA] [1]. See Figure 8 – 6.

8. Highlight the appropriate alternative hypothesis and press [ENTER]. We will highlight $<\mu_0$ for our example. Your screen should appear as in Figure 8 – 6.

9. Highlight Calculate and press [ENTER]. Your screen will appear as in Figure 8 – 7.

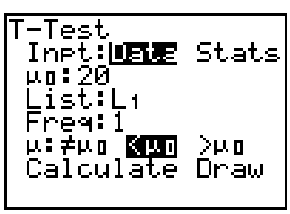

Figure 8 – 6

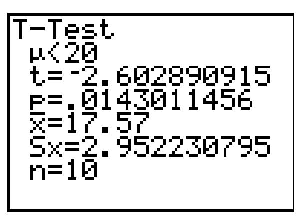

Figure 8 – 7

In Figure 8 – 7, we see the test statistic is t = -2.6029 and the p-value is 0.0143 when rounded to 4 decimal places.

Remember: If p-value $\leq \alpha$ then reject H_0; otherwise do not reject H_0.

Our p-value is 0.0143, which is larger than the specified significance level of 0.01. Therefore we do not reject the null hypothesis. Thus, there is not sufficient evidence at $\alpha = .01$ to conclude this new type of engine meets the standard that emissions be less than 20 ppm.

Large Sample Test of Hypothesis about a Population Proportion

Examples 8.9 & 8.10 A Hypothesis Test for *p* – Proportion of Defective Batteries

Problem: The reputation (and hence sales) of many businesses can be severely damaged by shipments of manufactured items that contain a large percentage of defectives. For example, a manufacturer of alkaline batteries may want to be reasonably certain that less than 5% of its batteries are defective. Suppose 300 batteries are randomly selected from a very large shipment; each is tested and 10 defective batteries are found. Does this outcome provide sufficient evidence to conclude that the proportion of defective batteries in the entire shipment is less than 0.05? Use $\alpha = 0.01$.

Solution: Again, on the calculator this problem will be worked quite differently than it is by hand yet the results will be the same. Start by noting the given information. n = 300, x = 10, and $\alpha = 0.01$.

Let p denote the proportion of all batteries that are defective. Then the null and alternative hypotheses are

H_0: p = 0.05 (Proportion of defective batteries equals 0.05)
H_a: p < 0.05 (Proportion of defective batteries is less than 0.05)

Note that this is a left-tailed test.

1. Press STAT and arrow over to the TESTS menu.

2. Press 5 or arrow down to **5:1-PropZTest** and press ENTER. See Figure 8 – 8.

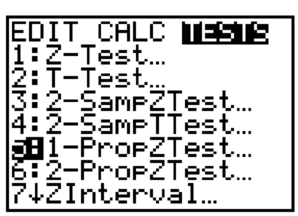

Figure 8 – 8

3. Enter your p_0, here, 0.05, and press ENTER. See Figure 8 – 9.

4. Enter your x, here 10, and press ENTER. See Figure 8 – 9.

5. Enter your n, here 300, and press ENTER. See Figure 8 – 9.

6. Highlight your alternative hypothesis, here $>p_0$ and press ENTER. Your screen should appear as in Figure 8 – 9.

7. Highlight Calculate and press ENTER. Your screen should appear as in Figure 8 – 10.

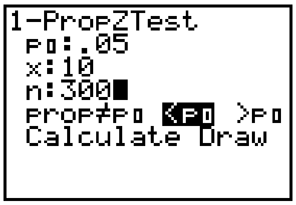

Figure 8 – 9

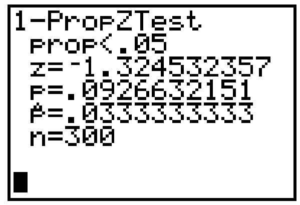

Figure 8 – 10

As can be seen from the Figure 8 – 10 the test statistic is z = -1.325 and the p-value is p = .0927 when rounded to four decimal places. The p-value of 0.0927 is not less than $\alpha = 0.01$. Therefore, do not reject H_0. In conclusion, we do not have sufficient evidence that the population proportion is less than 0.05.

Note: In Figure 8 – 10, we see the screen display also gives additional information. In particular:

The top line reminds you what test you are performing.
The second line is the alternative hypothesis (H_a).
The third line is the test statistic.
The fourth line is the sample proportion.
The bottom line is the sample size.