

Chapter 7

Inferences Based on a Single Sample

Confidence Interval for a Population Mean: Normal (Z) Statistic

Example 7.1 Estimating the Mean Hospital Length of Stay, σ Known

Problem: Consider the large hospital that wants to estimate the mean length of stay of its patients. The hospital randomly samples $n = 100$ of its patients and finds the sample mean length stay is 4.5 days. Also, suppose it is known that the standard deviation of the length of stay for all hospital patients is 4 days. Calculate a 95% confidence interval for the target parameter.

Solution: The sample size is large ($n = 100$) and σ is known (4 days). Therefore we can use the z-interval procedure.

There are two methods to construct a z-interval using the TI-83/84 Plus. One method is useful if you have the data values but have not computed the mean. The other method is useful if you do not have the data values but have the sample mean and sample size. In this example we do not have the data, but we have the sample mean and sample size.

1. Press **[STAT]** and arrow over to the TESTS menu. See Figure 7 – 1.
2. Number 7 is the ZInterval so either press **[7]** or arrow down to 7 and press **[ENTER]**. See Figure 7 – 1.

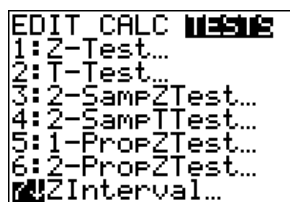


Figure 7 – 1

3. Notice that for **Inpt (input)** you have 2 choices:
 - If you choose **data** the calculator will read from a list.
 - If you choose **Stats** then you must enter the statistics yourself.

For this problem we have the sample mean and sample size so we will choose **Stats** and enter the given information. Therefore, highlight Stats and press **[ENTER]**. See Figure 7 – 2.

4. Enter your σ as 4.0, and press **[ENTER]**. See Figure 7 – 2.
5. Enter your \bar{x} as 4.5 and press **[ENTER]**. See Figure 7 – 2.

6. Enter your n as 100 and press **[ENTER]**. See Figure 7 – 2.
7. Enter the C-Level as 0.95 and press **[ENTER]**. Remember to enter the confidence interval as a decimal. Your screen should appear as in Figure 7 – 2.
8. Highlight Calculate and press **[ENTER]**. Your answer will be displayed as in Figure 7 – 3.

```
ZInterval
Inpt:Data STAT
σ:4
x̄:4.5
n:100
C-Level:.95
Calculate
```

Figure 7 – 2

```
ZInterval
(3.716,5.284)
x̄=4.5
n=100
```

Figure 7 – 3

We can be 95% confident that the mean length of stay, μ , for all patients is somewhere between 3.716 days and 5.284 days.

Confidence Interval for a Population Mean: Student’s t-Statistic

Example 7.5 A Small Sample Confidence Interval for μ – Destructive Sampling

Problem: Some quality control experiments require destructive sampling in order to measure a particular characteristic of the product. The cost of destructive sampling often dictates the use of small samples. Suppose a manufacturer of printers for personal computers wishes to estimate the mean number of characters printed before the printhead fails. The printer manufacturer tests $n=15$ printheads and records the number of characters printed until failure for each. These 15 measurements (in millions of characters) are listed in Table 7 – 1.

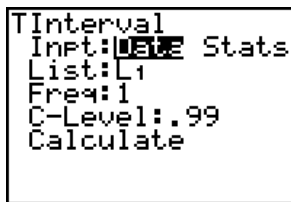
Table 7 – 1
Sample Data

Number of Characters (in Millions) for $n = 15$ Printheads				
1.13	0.92	1.32	1.48	1.18
1.55	1.25	0.85	1.20	1.22
1.43	1.36	1.07	1.33	1.29

Solution: For this problem we do not have the population standard deviation so will need to use the sample standard deviation to estimate the population standard deviation. Therefore, we will form a T-Interval for the population mean.

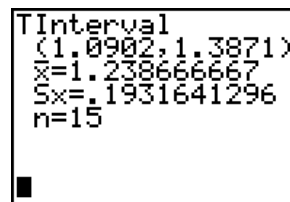
Like the z-interval, there are two ways to do a t-interval on the TI-83/84 Plus. One method is used when you have the data values. The other method is used when you have the sample mean and sample standard deviation.

1. With the data in List 1, press **[STAT]** and arrow over to the TESTS menu.
2. Number 8 is the TInterval so either press **[8]** or arrow down to 8 and press **[ENTER]**.
3. Notice that for **Inpt (input)** you have 2 choices:
 If you choose **data** the calculator will read from a list.
 If you choose **Stats** then you must enter the statistics yourself.
 For this problem we have the data so we will choose **Data** and enter the given information. Therefore, highlight Data and press **[ENTER]**. See Figure 7 – 4.
4. Press **[2nd]** L1 to enter List 1. See Figure 7 – 4.
5. Set the Freq: as 1. See Figure 7 – 4.
6. Set the C-Level as 0.99. Remember to enter the confidence level as decimal. Your screen should appear as in Figure 7 – 4.
7. Highlight Calculate and press **[ENTER]**. Your answer will be displayed as in Figure 7 – 5.



```
TInterval
Inpt: DATA Stats
List:L1
Freq:1
C-Level:.99
Calculate
```

Figure 7 – 4



```
TInterval
(1.0902, 1.3871)
x=1.238666667
Sx=.1931641296
n=15
```

Figure 7 – 5

Figure 7 – 5 we see the confidence interval is 1.0902 to 1.3871. We can be 99% confident that the mean number of characters printed before the printhead fails, μ , of all printers is somewhere between 1.0902 and 1.3871 million characters.

Large Sample Confidence Interval for a Population Proportion

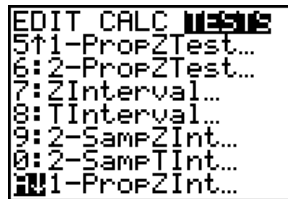
Example 7.7 Large Sample Confidence Interval for p – Proportion Optimistic About the Economy

Problem: Many public polling agencies conduct surveys to determine the current consumer sentiment concerning the state of the economy. In a random sample of 484 consumers, only 157 were optimistic about the state of the economy. Use a 90% confidence interval to estimate the proportion of all consumers who are optimistic about the economy.

Solution: For this problem we want a **Proportion-Interval**. We are given $x = 157$, and $n = 484$.

Recall: x is the number of successes, n is the sample size and **C-level** is confidence level.

1. Press **[STAT]** and arrow over to the TESTS menu. See Figure 7 – 6.
2. Press **[ALPHA]** A or arrow down to **A:1-PropZInt**. See Figure 7 – 6. Press **[ENTER]**.

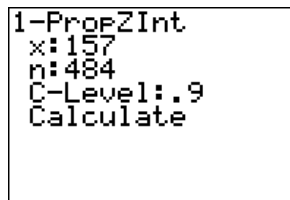


```

EDIT CALC TESTS
5:1-PropZTest...
6:2-PropZTest...
7:ZInterval...
8:TInterval...
9:2-SampZInt...
0:2-SampTInt...
1:1-PropZInt...
  
```

Figure 7 – 6

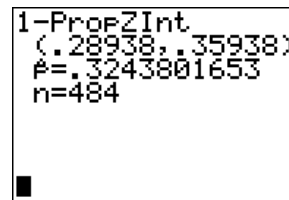
3. Enter your x -value, here 157, and press **[ENTER]**. See Figure 7 – 7.
4. Enter your n -value, here 484, and press **[ENTER]**. See Figure 7 – 7.
5. Enter the C-level, here 0.90, and press **[ENTER]**. Your screen should appear as in Figure 7 – 7.
6. Highlight Calculate and press **[ENTER]**. Your screen should appear as in Figure 7 – 8.



```

1-PropZInt
x:157
n:484
C-Level:.9
Calculate
  
```

Figure 7 – 7



```

1-PropZInt
(.28938, .35938)
P=.3243801653
n=484
  
```

Figure 7 – 8

From Figure 7 – 8 we see the 90% confidence interval is from 0.289 to 0.359 when rounded to three decimal places. We can be 90% confident that the percentage of all consumers who are confident about the economy is somewhere between 28.9% and 35.9%.