Chapter 6
Sampling Distributions

The Sampling Distribution of $\overline{x}$ and the Central Limit Theorem

Example 6.8 Application of the Central Limit Theorem – Testing a Manufacturer's Claim

Problem: A manufacturer of automobile batteries claims that the distribution of the lengths of its best battery has a mean of 54 months and a standard deviation of 6 months. Suppose a consumer group decides to check the claim by purchasing a sample of 50 batteries and subjecting them to tests that estimate the battery's life.

a) Assuming that the manufacturer's claim is true, describe the sampling distribution of the mean lifetime of a sample of 50 batteries.

b) Assuming that the manufacturer's claim is true, what is the probability that the consumer group's sample has a mean life of 52 or fewer months?

Solution:

a) The sampling distribution of the mean lifetime of a sample of 50 batteries will be distributed normally with $\mu_{\overline{x}} = \mu = 54$ and $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{50}} = 0.84853$.

b) Recall **normalcdf**(lowerbound, upperbound, μ, σ) is the format of the command. For our problem, the lowerbound is -1E99 , the upperbound is 52 and the mean and standard deviation are 54 and 0.84853, respectively.

1. Access the **normalcdf(** command by pressing [2nd] DISTR [2].

2. Your command should be **normalcdf**(-1E99,52,54,0.84853) [ENTER]. The command and its result are shown in Figure 6 – 1.

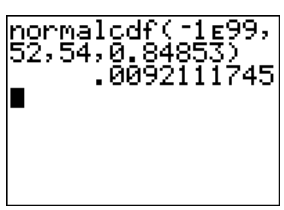

Figure 6 – 1

As can be seen from Figure 6 – 1 the probability that the consumer group will observe a mean of 52 or fewer months is 0.0092. The value in the textbook is calculated using the z-tables and therefore has a small rounding error.

Exercise 6.31 Application of the Central Limit Theorem

Problem: A random sample of n =100 observations is selected from a population with $\mu = 30$ and $\sigma = 16$.

a) Find $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$.
b) Describe the shape of the sampling distribution of the sample mean.
c) Find $P(\bar{x} \geq 28)$.
d) Find $P(22.1 \leq \bar{x} \leq 26.8)$.
e) Find $P(\bar{x} \leq 28.2)$.
f) Find $P(\bar{x} \geq 27.0)$.

Solution:

a) $\mu_{\bar{x}} = \mu = 30$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{100}} = 1.6.$

b) The sampling distribution of the sample mean will be distributed normally distributed with $\mu_{\bar{x}} = \mu = 30$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{100}} = 1.6.$

c) Recall **normalcdf(**lowerbound, upperbound, μ, σ) is the format of the command. To find $P(\bar{x} \geq 28)$ the lowerbound is 28 and the upperbound is 1E99 and the mean and standard deviation are 30 and 1.6, respectively.

1. Access the **normalcdf(** command by pressing [2nd] DISTR [2].

2. Your command should be **normalcdf(**28,1E99,30,1.6) [ENTER]. The command and its result are shown in Figure 6 – 2.

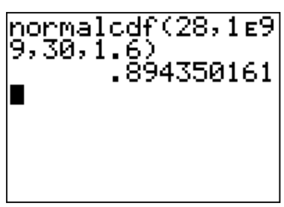

Figure 6 – 2

As can be seen from Figure 6 – 2 the probability that the sample mean will be 28 or more is $P(\bar{x} \geq 28) = 0.8944$. The value in the textbook is calculated using the z-tables and therefore has a small rounding error.

d) Recall **normalcdf(**lowerbound, upperbound, μ, σ) is the format of the command. To find $P(22.1 \leq \bar{x} \leq 26.8)$ the lowerbound is 22.1 and the upperbound is 26.8 and the mean and standard deviation are 30 and 1.6, respectively.

1. Access the **normalcdf(** command by pressing [2nd] DISTR [2].

2. Your command should be **normalcdf(**22.1,26.8,30,1.6) ENTER. The command and its result are shown in Figure 6 – 3.

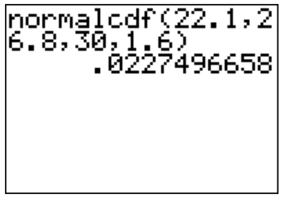

Figure 6 – 3

As can be seen from Figure 6 – 3 the probability that the sample mean will be between 22.1 and 26.8 is $P(22.1 \le \bar{x} \le 26.8) = 0.0227$. The value in the textbook is calculated using the z-tables and therefore has a small rounding error.

e) Recall **normalcdf(**lowerbound, upperbound, μ, σ) is the format of the command. To find $P(\bar{x} \le 28.2)$ the lowerbound is -1E99 and the upperbound is 28.2 and the mean and standard deviation are 30 and 1.6, respectively.

1. Access the **normalcdf(** command by pressing 2nd DISTR 2.

2. Your command should be **normalcdf(**-1E99,28.2,30,1.6) ENTER. The command and its result are shown in Figure 6 – 4.

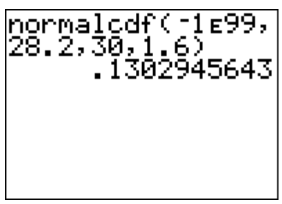

Figure 6 – 4

As can be seen from Figure 6 – 4 the probability that the sample mean will be at most 28.2 is $P(\bar{x} \le 28.2)$ =0.1303. The value in the textbook is calculated using the z-tables and therefore has a small rounding error.

f) Recall **normalcdf(**lowerbound, upperbound, μ, σ) is the format of the command. To find $P(\bar{x} \ge 27.0)$ the lowerbound is 27 and the upperbound is 1E99 and the mean and standard deviation are 30 and 1.6, respectively.

1. Access the **normalcdf(** command by pressing 2nd DISTR 2.

2. Your command should be **normalcdf(**27,1E99,30,1.6) ENTER. The command and its result are shown in Figure 6 – 5.

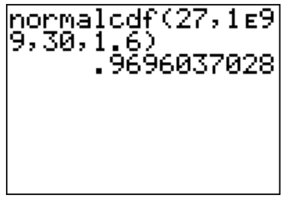

Figure 6 – 5

As can be seen from Figure 6 – 5 the probability that the sample mean will be 27 or more is $P(\bar{x} \geq 27.0) = 0.9696$. The value in the textbook is calculated using the z-tables and therefore has a small rounding error.

Exercise 6.45 Is Exposure to a Chemical in Teflon-coated Cookware Hazardous?

Problem: Perfluorooctanoic acid (PFOA) is a chemical used in Teflon-coated cookware to prevent food from sticking. The Environmental Protection Agency (EPA) is investigating the potential risk of PFOA as a cancer-causing agent (*Science News Online*, August 27, 2005). It is known that the blood concentration of PFOA in the general population has a mean of $\mu = 6$ parts per billion (ppb) and a standard deviation of $\sigma = 10$ ppb. *Science News Online* reported on tests for PFOA exposure conducted on a sample of 326 people who live near DuPont's Teflon-making Washington (West Virginia) Works facility.
a) What is the probability that the average blood concentration of PFOA in the sample is greater than 7.5 ppb?
b) The actual study resulted in $\bar{x} = 300$ ppb. Use this information to make an inference about the true mean (μ) PFOA concentration for the population that lives near DuPont's Teflon facility.

Solution: a) The sampling distribution of the mean blood concentration of PFOA in a sample of 326 people will be normally distributed with $\mu_{\bar{x}} = \mu = 6$ and

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{326}} = 0.55385.$$

Recall **normalcdf(**lowerbound, upperbound, μ, σ) is the format of the command. For our problem, the lowerbound is 7.5 , the upperbound is 1E99 and the mean and standard deviation are 6 and 0.55385, respectively.

1. Access the **normalcdf(** command by pressing [2nd] DISTR [2].

2. Your command should be **normalcdf(**7.5,1E99,6,0.55385) [ENTER]. The command and its result are shown in Figure 6 – 6.

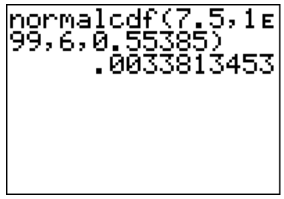

Figure 6 – 6

As can be seen from Figure 6 – 6 the probability that the average blood concentration of PFOA in the sample is greater than 7.5 ppb is 0.0034. The value in the textbook is calculated using the z-tables and therefore has a small rounding error.

b) The sampling distribution of the mean blood concentration of PFOA in a sample of 326 people will be normally distributed with $\mu_{\bar{x}} = \mu = 6$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{326}} = 0.55385$. A sample mean of 300 ppb corresponds to a z-score of 530.8. Thus, a sample mean of 300 ppb should virtually never occur by natural sampling variability. It is very highly probable that the true mean (μ) PFOA concentration for the population of people who live near the DuPont's Teflon Facility exceeds the mean of 6 ppb for the general population.