

# Chapter 5

## Continuous Random Variables

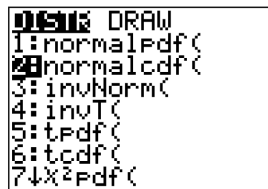
### The Normal Distribution

#### Example 5.3 Using the Standard Normal Table to find $P(-z_0 < z < z_0)$

**Problem:** Find the probability that the standard normal random variable  $z$  falls between  $-1.33$  and  $1.33$ . That is, find  $P(-1.33 < z < 1.33)$ .

**Solution:** We need to find the area under the standard normal curve from  $-1.33$  to  $1.33$ . To do this we will use the **normalcdf**( function on the TI-83/84 Plus.

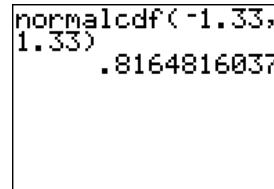
1. Press **[2nd]** **DISTR** to access the distribution menu. See Figure 5 – 1.
2. Select **normalcdf**( by pressing **[2]** or arrow down to **2:normalcdf**( and pressing **[ENTER]**.
3. The format of this command is **normalcdf**(lowerbound, upperbound,  $\mu$ ,  $\sigma$ ) with the default mean of 0 and the default standard deviation of 1. This means that when working with the standard normal distribution, it is not necessary to enter  $\mu = 0$  and  $\sigma = 1$ . Thus our command becomes **normalcdf**(-1.33,1.33) **[ENTER]**. See Figure 5 – 2 for the command and the answer.



```

DISTR DRAW
1:normalPdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tPdf(
6:tcdf(
7:χ²Pdf(
  
```

Figure 5 – 1



```

normalcdf(-1.33,
1.33)
      .8164816037
  
```

Figure 5 – 2

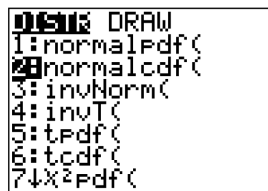
As can be seen from Figure 5 – 2, the probability is  $P(-1.33 < z < 1.33) = 0.8164816037$ . This means that for a normally distributed random variable, the probability of being within 1.33 standard deviations of the mean is approximately 0.8164.

**Example 5.4 Using the Standard Normal Table to find  $P(z > z_0)$** 

**Problem:** Find the probability that a standard normal random variable exceeds 1.64; that is, find  $P(z > 1.64)$ .

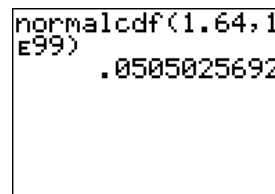
**Solution:** We need to find the area under the curve from 1.64 to  $\infty$ . To do this we will use the **normalcdf**( function on the TI-83/84 Plus.

1. Press  $\boxed{2\text{nd}}$  DISTR to access the distribution menu. See Figure 5 – 3.
2. Select **normalcdf**( by pressing  $\boxed{2}$  or arrow down to **2:normalcdf**( and pressing  $\boxed{\text{ENTER}}$ .
3. The format of this command is **normalcdf**(lowerbound, upperbound,  $\mu, \sigma$ ) with the default mean of 0 and the default standard deviation of 1. This means that when working with the standard normal distribution, it is not necessary to enter  $\mu = 0$  and  $\sigma = 1$ . Because the TI-83/84 Plus does not understand  $\infty$  we will use 1 times 10 raised to the 99th power or 1E99. Thus our command becomes **normalcdf**(1.64,1E99)  $\boxed{\text{ENTER}}$ . Note: The E is obtained by pressing  $\boxed{2\text{nd}}$  EE. See Figure 5 – 4 for the command and the answer.



A screenshot of the TI-83/84 Plus calculator's DISTR menu. The menu options are: 0:SD, DRAW, 1:normalPdf(, 2:normalcdf(, 3:invNorm(, 4:invT(, 5:tPdf(, 6:tcdf(, and 7:↓X²Pdf(.

Figure 5 – 3



A screenshot of the TI-83/84 Plus calculator showing the command normalcdf(1.64,1E99) entered and the result .0505025692 displayed.

Figure 5 – 4

From Figure 5 – 4 we see the probability is  $P(z > 1.64) = 0.0505025692$ . This means that for a normally distributed random variable, the probability of being more than 1.64 standard deviations above the mean is approximately 0.0505.

**Example 5.5 Using the Standard Normal Table to find  $P(z < z_0)$** 

**Problem:** Find the probability that a standard normal random variable lies to the left of 0.67; that is, find  $P(z < 0.67)$ .

**Solution:** We need to find the area under the curve from  $-\infty$  to 0.67. To do this we will use the **normalcdf**( function on the TI-83/84 Plus.

1. Press  $\boxed{2\text{nd}}$  DISTR to access the distribution menu. See Figure 5 – 5.

2. Select **normalcdf**( by pressing  $\boxed{2}$  or arrow down to **2:normalcdf**( and pressing  $\boxed{\text{ENTER}}$ .
3. The format of this command is **normalcdf**(lowerbound, upperbound,  $\mu, \sigma$ ) with the default mean of 0 and the default standard deviation of 1. This means that when working with the standard normal distribution, it is not necessary to enter  $\mu = 0$  and  $\sigma = 1$ . The TI-83/84 Plus does not understand  $-\infty$  therefore we will use  $-1$  times 10 raised to the 99th power or -1E99. Thus our command becomes **normalcdf**(-1E99, 0.67)  $\boxed{\text{ENTER}}$ .  
 Note: The E is obtained by pressing  $\boxed{2\text{nd}}$  EE. See Figure 5 – 6 for the command and the answer.

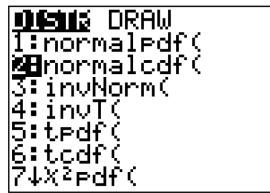


Figure 5 – 5

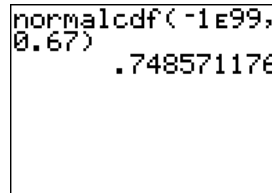


Figure 5 – 6

Figure 5 – 6 shows that the probability is  $P(z < 0.67) = 0.748571176$ . This means that for a normally distributed random variable, the probability of being below a z-score of 0.67 is approximately 0.7486.

### Example 5.7 Finding a Normal Probability – Cell Phone Application

**Problem:** Assume that the length of time,  $x$ , between charges of a cellular phone is normally distributed with a mean of 10 hours and a standard deviation of 1.5 hours. Find the probability that the cell phone will last between 8 and 12 hours between charges.

**Solution:** Recall **normalcdf**(lowerbound, upperbound,  $\mu, \sigma$ ) is the format of the command. For our problem, the lowerbound is 8, the upperbound is 12 and the mean and standard deviation are 10 and 1.5, respectively.

1. Access the **normalcdf**( command by pressing  $\boxed{2\text{nd}}$  DISTR  $\boxed{2}$ .
2. Your command should be **normalcdf**(8,12,10,1.5)  $\boxed{\text{ENTER}}$ . The command and its result are shown in Figure 5 – 7.

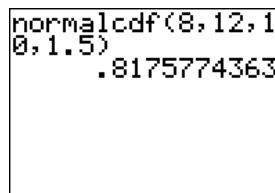


Figure 5 – 7

Therefore we can say that the probability the cell phone lasts between 8 and 12 hours is 0.8176. The textbook gives the answer to this example as 0.8164. This is because the textbook must round the z-scores to two decimal places to match the tables and the areas are rounded to four decimal places in the tables. The TI-83/84 Plus does not round the z-scores or the areas. Therefore, answers will almost always be slightly different when using this method. To obtain answers closer to the text book, compute the z-scores first, then obtain the areas using the z-scores.

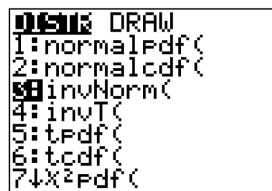
**Using Normal Tables in Reverse** Up until now, we have used the TI-83/84 Plus to find the areas under the standard normal curve. Now we will use the TI-83/84 Plus to find the z-scores corresponding to a specified area under the standard normal curve.

### Example 5.9 Using Normal Tables in Reverse

**Problem:** Find the value of  $z$  – call it  $z_0$  – in the standard normal distribution that will be exceeded only 10% of the time. That is, find  $z_0$  such that  $P(z \geq z_0) = 0.10$ .

**Solution:** We will use the **invNorm**( function to find the z-score.

1. Press **[2nd]** **DISTR** to access the distribution menu. See Figure 5 – 8.
2. Select **invNorm**( by pressing **[3]** or arrow down to **3:invNorm**( and pressing **[ENTER]**.
3. The format of this command is **invNorm**(area to the left,  $\mu$ ,  $\sigma$ ). Like the **normalcdf**( command the default mean and standard deviation are 0 and 1 respectively. This means that when working with the standard normal distribution, it is not necessary to enter  $\mu = 0$  and  $\sigma = 1$ . The format of this command requires that the area to the left of the score be entered. If the area to the right is 0.10 then the area to the left is  $1 - 0.10 = 0.90$ . This means that our desired z-value has an area 0.90 to its left under the normal curve. Thus, our command is **invNorm**(0.90) **[ENTER]**. The command and its output are shown in Figure 5 – 9.

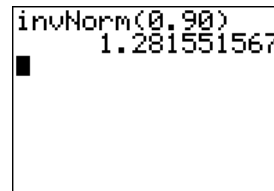


```

DISTR DRAW
1:normalPdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tPdf(
6:tcdf(
7:χ²Pdf(

```

Figure 5 – 8



```

invNorm(0.90)
1.281551567

```

Figure 5 – 9

As can be seen from the Figure 5 – 9, the z-score with an area of 0.10 to the right is  $z = 1.281551567$ . The TI-83/84 Plus computes the z-score to 9 decimal places. Therefore your answers may differ slightly from the textbook even when rounding the final answer to two decimal places.

**Example 5.11 Using Normal Tables in Reverse – College Entrance Exam Application**

**Problem:** Suppose the scores  $x$  on a college entrance examination are normally distributed with a mean of 550 and a standard deviation of 100. A certain prestigious university will consider for admission only those applicants whose scores exceed the 90<sup>th</sup> percentile of the distribution. Find the minimum score that an applicant must achieve in order to receive consideration for admission to the university.

**Solution:** Again we are given area under the curve, but this time we are asked for the actual value and not the z-score.

The 90th percentile is the examination score that is higher than those of 90% of all people. This means that our desired  $x$ -value has an area 0.90 to its left under the normal curve associated with examination scores. To find this value, use **invNorm**(

1. Press  $\boxed{2\text{nd}} \boxed{\text{DISTR}} \boxed{3}$  to access the command.
2. Remember the format is **invNorm**(area to the left,  $\mu, \sigma$ ). Our command is **invNorm**(0.90,550,100). The command and its result are shown in Figure 5 – 10.

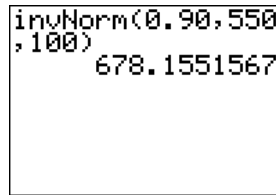


Figure 5 – 10

As can be seen from Figure 5 – 10 the minimum score on the entrance exam is 678.1551567. Since you probably can't score this we would say the minimum score is 678. Once again, the calculator gives a slightly different answer than the textbook because the table must round the z-score.

**Descriptive Measures for Assessing Normality**

**Exercise 5.59d Constructing a Normal Probability Plot**

**Problem:** Generate a normal probability plot for the data in Table 5 – 1, and use it to assess whether the data are approximately normal.

<b>Table 5 – 1</b>	32	48	25	135	53	37	5	39	213	165
	109	40	1	146	39	25	21	66	64	57
	197	60	112	10	155	134	301	304	107	82
	35	81	60	95	401	308	180	3	200	59

## 5-6 Chapter 5

**Solution:** Enter the data from Table 5 – 1 into List 1. The TI-83/84 Plus has a built in normal probability plot function. When using this function, the order of the data does not matter.

1. Press  $\boxed{2nd}$  STAT PLOT.
2. Be sure all plots except Plot 1 are turned off and then press  $\boxed{ENTER}$ .
3. Highlight On and press  $\boxed{ENTER}$ . See Figure 5 – 11.
4. For Type, choose the last one  $\swarrow$  by moving over to it and pressing  $\boxed{ENTER}$ . See Figure 5 – 11.
5. Enter the data list as List 1 by moving to Data List and pressing  $\boxed{2nd}$  L1. See Figure 5 – 11.
6. Choose the data axis to be X by highlighting X and pressing  $\boxed{ENTER}$ . See Figure 5 – 11.
7. Choose the mark you prefer to use by highlighting it and pressing  $\boxed{ENTER}$ . See Figure 5 – 11.

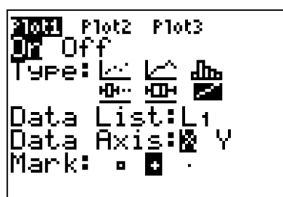


Figure 5 – 11

8. Set your window based on your data. Look at your data to pick an appropriate Xmin and Xmax. For this example, set Xmin at 0, Xmax at 410 and Xscl at 10.
9. The Ymin and Ymax are a little trickier. If they are too far apart, it can change the appearance of your graph. In general, if your sample size is less than 25 you can use -2 and 2. If your sample size is between 25 and 50, you can use -2.5 and 2.5. Your Yscl can be set at 0.5.

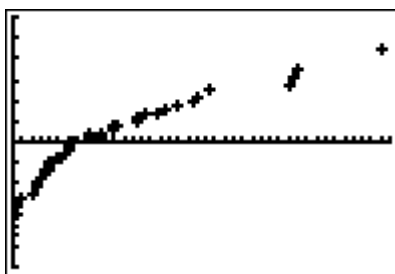


Figure 5 – 12

There appears to be significant curvature in the normal probability plot seen in Figure 5 – 12. Therefore, we conclude that the sample does not appear to come from a population that is normally distributed.