Chapter 4
Discrete Random Variables

Expected Values of Discrete Random Variables

Example 4.8 Finding μ and σ – Skin Cancer Treatment

Problem: Medical research has shown that a certain type of chemotherapy is successful 70% of the time when used to treat skin cancer. Suppose five skin cancer patients are treated with this type of chemotherapy and let x equal the number of successful cures out of the five. The probability distribution for the number x of successful cures out of 5 is given in Table 4 – 1.

Table 4 – 1	x	0	1	2	3	4	5
	p(x)	0.002	0.029	0.132	0.309	0.360	0.168

a) Find $\mu = E(x)$.

b) Find $\sigma = \sqrt{E\left[(x-\mu)^2\right]}$.

Solution:

1. Enter the x values in L_1 and the corresponding probabilities in L_2. See Figure 4 – 1. Note: For all data given in a probability distribution or listed as two or more variables it is necessary to enter the data in the EXACT order it is given.

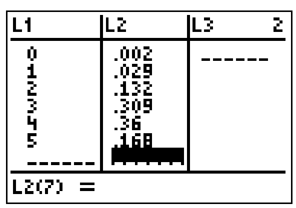

Figure 4 – 1

2. Press STAT and arrow over to the CALC menu. Press 1 or press ENTER for **1-Var Stats**. This will now appear on your home screen.

3. Enter the x list first and then the frequency list. For this example, we will use L_1 and L_2. Be sure to separate the names of the lists with a comma (key above the 7 key). Your screen should appear as in Figure 4 – 2.

4. Press ENTER. The results will be displayed as in Figure 4 – 3. Note that the TI-83/84 Plus only uses one symbol for the mean so our μ is listed as $\bar{x}$.

Figure 4 – 2

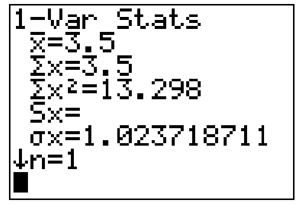

Figure 4 – 3

In Figure 4 – 3 we see that $\mu = E(x) = 3.5$ and $\sigma = \sqrt{E\left[(x-\mu)^2\right]} = 1.02$.

The Binomial Random Variable

Example 4.10 Deriving the Binomial Probability Distribution – Passing a Physical Fitness Exam

Problem: The Heart Association claims that only 10% of U.S. adults over 30 years of age meet the minimum requirements established by the President's Council on Fitness, Sports, and Nutrition. Suppose four adults are randomly selected and each is given the fitness test.
a) Find the probability that none of the four adults passes the test.
b) Find the probability that three of the four adults pass the test.

Solution: a) The TI-83/84 Plus has the binomial distribution built into it. It can find the probability of an individual x value or the entire distribution.

1. From the home screen, press 2nd DISTR to access the distribution menu. Your screen should appear as in Figure 4 – 4.

2. Press the zero key or arrow down to **0:binompdf(** and press ENTER. Your calculator will return to the home screen and display **binompdf(**.

3. The format of the binompdf command is **binompdf(**numtrials, p, x) with the x being optional. In the binompdf commad, p is the probability that an individual adult does not pass the fitness test. To find our probability that none of the four adults pass the fitness test, we will set up the command as **binompdf(**4,0.10,0) and then press ENTER. Your screen will appear as in Figure 4 – 5.

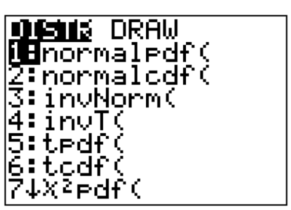

Figure 4 – 4

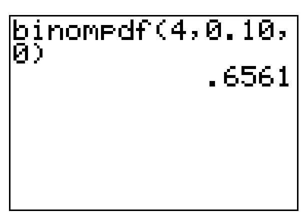

Figure 4 – 5

In Figure 4 – 5, we see the probability that none of the four adults passes the test is 0.6561.

Solution: b) The TI-83/84 Plus has the binomial distribution built into it. It can find the probability of an individual x value or the entire distribution.

1. From the home screen, press [2nd] DISTR to access the distribution menu. Your screen should appear as in Figure 4 – 6.

2. Press the zero key or arrow down to **0:binompdf(** and press [ENTER]. Your calculator will return to the home screen and display **binompdf(**.

3. The format of the binompdf command is **binompdf(**numtrials, p, x) with the x being optional. In the binompdf commad, p is the probability that an individual adult does not pass the fitness test. To find our probability that none of the four adults pass the fitness test, we will set up the command as **binompdf(**4,0.10,3) and then press [ENTER]. Your screen will appear as in Figure 4 – 7.

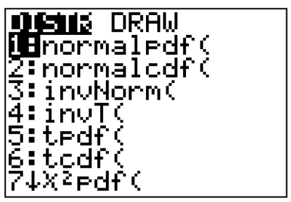

Figure 4 – 6

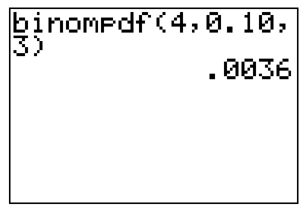

Figure 4 – 7

In Figure 4 – 7 we see the probability that three of the four adults will pass the test is 0.0036.

The Poisson Random Variable

Example 4.14 Finding Poisson Probabilities – Whale Sightings

Problem: Ecologists often use the number of reported sightings of a rare species of animal to estimate the remaining population size. For example, suppose the number, x, of reported sightings per week of the blue whale is recorded. Assume that x has (approximately) a Poissson probability distribution. Furthermore, assume that the average number of weekly sightings is 2.6.

a) Find the mean and standard deviation of x, the number of blue-whale sightings per week.
b) Find the probability that fewer than two sightings are made during a given week.
c) Find the probability that more than five sightings are made during a given week.
d) Find the probability that exactly five sightings are made during a given week.

Solution: a) The mean and variance of a Poisson random variable are both equal to λ. For this example, the mean is 2.6 so $\mu = \lambda = 2.6$ and $\sigma^2 = \lambda = 2.6$. The standard deviation of this Poisson distribution is $\sigma = \sqrt{2.6} = 1.61245$.

Solution: b) We will use the poissoncdf to find the probability of fewer than 2 sightings in a week.

1. From the home screen, press [2nd] DISTR to access the distribution menu. Press [ALPHA] D or arrow down to **D:poissoncdf(** Your screen should appear as in Figure 4 – 8.

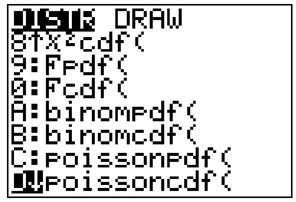

Figure 4 – 8

2. Press [ENTER]. Your calculator will return to the home screen and display **poissoncdf(**.

3. The format for this command is **poissoncdf(** μ, x). For the Poisson distribution, μ = λ. The command **poissoncdf(** μ, x) calculates the cumulative probability from 0 to x, inclusive. For this example, we want fewer than 2 sighting so we want the cumulative probability from 0 to 1. The command will be **poissoncdf(**2.6, 1) [ENTER].
See Figure 4 – 9.

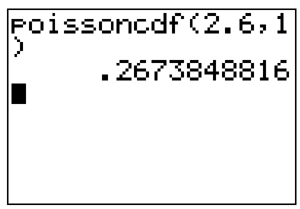

Figure 4 – 9

In Figure 4 – 9 we see the probability that fewer than two sightings of blue-whales are made during a given week is 0.2674.

Solution: c) We will use the poissoncdf to find the probability of more than 5 sightings in a week.

1. From the home screen, press [2nd] DISTR to access the distribution menu. Press [ALPHA] D or arrow down to **D:poissoncdf(** Your screen should appear as in Figure 4 – 10.

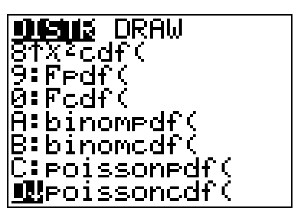

Figure 4 – 10

2. Press [ENTER]. Your calculator will return to the home screen and display **poissoncdf(**.

3. The format for this command is **poissoncdf(** μ, x). For the Poisson distribution, $\mu = \lambda$. The command **poissoncdf(** μ, x) calculates the cumulative probability from 0 to x, inclusive. For this example, we want more than 5 sightings. To find this, we will use the complementation rule. Thus we will be finding $P(x > 5) = 1 - P(0 \le x \le 5)$ The command will be 1 - **poissoncdf(**2.6, 5). Then press ENTER. See Figure 4 – 11.

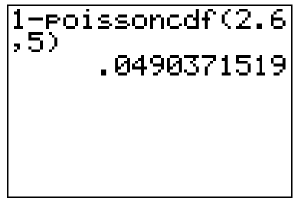

Figure 4 – 11

In Figure 4 – 11 we see the probability that more than five sightings of blue-whales are made during a given week is 0.04904.

Solution: d) We will use the poissonpdf to find the probability of exactly 5 sightings in a week.

1. For format for this command is poissonpdf(μ ,x). The command poissonpdf(μ ,x) calculates the probability of exactly x sightings in a week. From the home screen, press 2nd DISTR to access the distribution menu. Press ALPHA C or arrow down to **C:poissonpdf(** and press ENTER. See Figure 4 – 12. Your calculator will return to the home screen and display **poissonpdf(**.

2. The format for this command is **poissonpdf(**μ, x). For the Poisson distribution, $\mu = \lambda$. For this example, our command will be **poissonpdf(**2.6, 5) ENTER. See Figure 4 – 13.

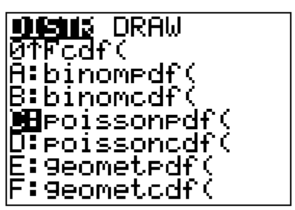

Figure 4 – 12

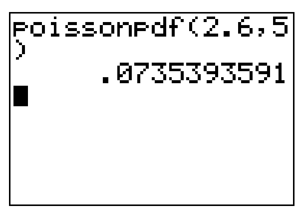

Figure 4 – 13

In Figure 4 – 13 we see the probability that exactly five sightings are made during a given week is 0.0735.