

## Extended Application / Cavities and Restoration

K. H. Lu of the University of Oregon Dental School has applied Markov chains to analyze the progress of teeth from a healthy state to having cavities to being restored through a filling or becoming so decayed as to require removal.\*

Dental records of 184 grade and high school students were observed at six-month intervals (a total of 1080 records). The state of the upper second premolar of each student was recorded as a five-digit number. To interpret this number, consider the tooth being roughly shaped like a box, with one side fastened to the gum. There are five sides of the tooth where cavities can occur, and each digit of the five-digit number refers to one of these sides. The first digit referred to the occlusal (or chewing) surface, the second to the mesial surface (the surface closest to the front of the mouth), the third to the distal surface (the surface farthest from the front of the mouth), the fourth to the lingual

surface (next to the tongue), and the fifth to the buccal surface (next to the cheek). A 0 was used to signify a healthy surface, a 1 for a decayed surface, and a 2 for a restored surface, in which the decayed area has been filled. Thus, (00000) signifies a healthy tooth and (20100) a tooth with a filling on the occlusal surface and a cavity on the distal surface, with the other three surfaces healthy.

The total number of possible states in this Markov chain is  $3^5 = 243$ , but only 18 of these states occurred among the students. In the initial analysis, all 9 states with a 2 in one of the digits were grouped together under the heading "restoration." This is an absorbing state, because once a surface has been filled, the filling stayed there for the duration of the study. Another absorbing state is (11111); once the tooth reaches this point, it is lost.

Lu found the following transition matrix.

	00000	00100	01000	01100	10000	10100	11000	11100	Restoration	11111
00000	.84853	.07003	.03420	.01140	.01791	.00326	0	.00326	.01141	0
00100	0	.74725	0	.18681	0	.02198	0	0	.04396	0
01000	0	0	.70312	.26563	0	0	.03125	0	0	0
01100	0	0	0	.90510	0	0	0	.03650	.05840	0
10000	0	0	0	0	.71430	.17857	0	.03571	.03571	.03571
10100	0	0	0	0	0	.75000	0	.25000	0	0
11000	0	0	0	0	0	0	.50000	.50000	0	0
11100	0	0	0	0	0	0	0	.90000	.05000	.05000
Restoration	0	0	0	0	0	0	0	0	1	0
11111	0	0	0	0	0	0	0	0	0	1

Using a computer, he found the fundamental matrix to be as follows.

6.601967	1.829222	.760534	6.522655	.413865	.542530	.047533	4.337776
0	3.956479	0	7.788301	0	.347854	0	3.712363
0	0	3.368364	9.428226	0	0	.210523	4.493915
0	0	0	10.537407	0	0	0	3.846153
0	0	0	0	3.500175	2.500105	0	7.500173
0	0	0	0	0	4.000000	0	9.999998
0	0	0	0	0	0	2.000000	9.999998
0	0	0	0	0	0	0	9.999998

\*From K. H. Lu, "A Markov Chain Analysis of Caries Process with Consideration for the Effect of Restoration," *Archives of Oral Biology* 13, 1968, pp. 1119–32.



From the fundamental matrix and the transition matrix, it is possible to explore what happens to the students' teeth as time progresses. For example, Lu concluded from the small probabilities of entering the restoration state that the group was receiving poor dental care, and that only when cavities had done considerable damage to the teeth were restorative measures sought. Further evidence for the lack of dental care was the fact that the probability of going from state (01000), (11000), or (10100) to the restoration state is zero; students in these states did not receive professional care until more damage to the tooth had occurred. Other aspects of this study are explored in the exercises.

## Exercises



1. Use a computer to verify the fundamental matrix given in the text.
2. Given a student with a healthy tooth, use the fundamental matrix to approximate the amount of time until the student has some tooth decay. [*Hint*: Find the expected number of times state (00000) is visited, and use the fact that each time represents 6 mo.]
3. Compute  $FR$ . (*Note*: A computer may be helpful here.)
4. Use your answer from Exercise 3 to find the probability that a healthy tooth is eventually lost.
5. For one of the five surfaces, once decay had occurred, there was a .5 probability that the tooth would be lost. Use your answer from Exercise 3 to find which surface that is.